D-branes, Symplectomorphisms and Noncommutative Gauge Theories

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It is shown that the dual of the double compactified D=11 Supermembrane and a suitable compactified D=10 Super 4D-brane with nontrivial wrapping on the target space may be formulated as noncommutative gauge theories. The Poisson bracket over the world-volume is intrinsically defined in terms of the minima of the hamiltonian of the theory, which may be expressed in terms of a non degenerate 2-form. A deformation of the Poisson bracket in terms of the Moyal brackets is then performed. A noncommutative gauge theory in terms of the Moyal star bracket is obtained. It is shown that all these theories may be described in terms of symplectic connections on symplectic fibrations. The world volume being its base manifold and the (sub)group of volume preserving diffeomorphisms generate the symplectomorphisms which preserve the (infinite dimensional) Poisson bracket of the fibration.

1. Introduction

The formulation of D-brane theories in the presence of constant antisymmetric background fields and its relation to noncommutative gauge theories has recently attracted a lot of interest [1]-[22]. It may well be that a noncommutative formulation of the D=11 Supermembrane and the Super M5-brane may indeed improve the understanding of the quantum aspects of these theories. The spectrum of the D=11 Supermembrane on a Minkowski target space was shown to be continuous from zero to infinite [23]. However not much it is known about the spectrum of the theory when the target space is compactified [24]-[26]. Even less it is known about the spectrum of the M5-brane. Nevertheless, one may extrapolate some known aspects from the Supermembrane case since both theories are U-dual. The covariant formulation of the M5-brane was found in [27] and [28]. However the analysis of its physical hamiltonian, the existence of singular configurations, the topological instabilities and related problems have not yet been discussed in a conclusive way. It is possible that a formulation of these theories in terms of a noncommutative geometry may allow an improvement in their analysis.

We describe in this talk a general approach to reach that formulation. The first step is to introduce a symplectic geometry intrinsic to the theory. This may be done when the target space is suitably compactified. The non degenerate closed 2-form associated to the symplectic geometry may be obtained in a general way from the analysis of the Born-Infield action. We discuss this problem in section 2. The second step in the construction is to obtain the hamiltonian of the D-brane with non trivial wrapping on the target space. We perform this construction for the double compactified D=11 Supermembrane [29], [30] and the compactified 4 D-brane in 10 dimensions. It turns out that the minima of these hamiltonians are described by the dual configurations introduced in [31] and in section 2. The final step is to introduce the geometrical objects describing the noncommutative formulation. This is done in terms of a symplectic fibration and a symplectic

connection over it. We also consider deformations of the brackets introduced in these theories, allowing a construction of noncommutative gauge theories in terms of the usual Moyal star product. There is a precise one to one correspondence in the sense of Kontsevich, between the original theories and their deformations.

2. The dual configurations

The Born-Infeld theory formulated over a Riemannian manifold M may be described by the following D dimensional action

$$S(A) = \int_M \left(\sqrt{\det (g_{ab} + bF_{ab})} - \sqrt{g} \right) d^D x \qquad (1)$$

where g_{ab} is an external euclidean metric over the compact closed manifold M. F_{ab} are the components of the curvature of connection 1-form A over a $U(1)$ principle bundle on $M.b$ is a constant parameter.

We may express the $\det (g_{ab} + bF_{ab})$, using the general formula obtained in [31], as

$$\begin{aligned} \det (g_{ab} + bF_{ab}) &= g \sum_{m=0}^{n} a_m b^{2m} * [\mathbf{P}_m \wedge *\mathbf{P}_m] \\ &\equiv gW, \end{aligned} \qquad (2)$$

where

$$\mathbf{P}_m \equiv \underbrace{F \wedge \ldots \wedge F}_{m} \qquad (3)$$

a_m are known constants, see [31].

The first variation of (1) is given by

$$\delta S(A) = \int_M W^{-\frac{1}{2}} \sum_m m a_m b^{2m} d\delta A \wedge \mathbf{P}_{m-1} \wedge *\mathbf{P}_m \qquad (4)$$

which yields the following field equations

$$\sum_m m a_m b^{2m} \mathbf{P}_{m-1} \wedge d\left(W^{-\frac{1}{2}} *\mathbf{P}_m \right) = 0. \qquad (5)$$

We introduce now a set $\mathcal{A}$ of $U(1)$ connection 1-forms over M [31]. They are defined by the following conditions,

$$*\mathbf{P}_m (A) = k_m \mathbf{P}_{n-m} (A) \qquad , m = 0, \ldots, n, \qquad (6)$$

where $n = \frac{D}{2}$,i.e we assume the dimension D of M to be an even natural number. (6) is the condition that the Hodge dual transformation maps the set $\{\mathbf{P}_m \ , \quad m = 0, \ldots, n\}$ into itself.

We observe that these connections, if they exist in a $U(1)$ principle bundle over M, are solutions of the field equations (5).

In fact, (6) implies

$$* [\mathbf{P}_m \wedge *\mathbf{P}_m] = k_m * [\mathbf{P}_m \wedge \mathbf{P}_{n-m}] = k_m * \mathbf{P}_n \qquad (7)$$

but from (6), for $m = n$, we obtain

$$* \mathbf{P}_n = k_n \qquad (8)$$

which is constant. We thus have, for these connections,

$$W = \text{constant}. \qquad (9)$$

Finally, it results

$$d\left(W^{-\frac{1}{2}} *\mathbf{P}_m \right) = k_m W^{-\frac{1}{2}} d\left(\mathbf{P}_{n-m}\right) = 0, \qquad (10)$$

showing that (6), if they exits, define a set of solutions to the Born-Infeld field equations.

Let us analyse a particular case of (6). Let us consider $n = \frac{D}{2} = 1$. We then have

$$*\mathbf{P}_1 = *F = k_1. \qquad (11)$$

This solution represents a monopole connection over the $D = 2$ manifold M. When M is the sphere S_2, (11) defines the $U(1)$ connection describing the Dirac monopole on the Hopf fibring $S_3 \to S_2$. The constant k_1 is determined from the condition

$$\int_M F = 2\pi \times \text{integer} \qquad (12)$$

which is a necessary condition to be satisfied for a $U(1)$ connection, F being its curvature.

This solution was extended to $U(1)$ connections over Riemann surfaces of any genus in [32] . In [29] it was shown that they describe the minima of the hamiltonian of the double compactified $D = 11$ supermembrane dual.

3. Hamiltonian formulation

The hamiltonian formulation of the double compactified D=11 Supermembrane dual was obtained in [30]. Its hamiltonian density in the light

cone gauge is the following

$$\mathcal{H} = \frac{1}{2}\frac{1}{\sqrt{W}}\left(P^M P_M + det(\partial_a X^M \partial_b X_M)\right.$$
$$\left. + (\Pi^a_r \partial_a X^M)^2 + \frac{1}{4}(\Pi^a_r \Pi^b_s \epsilon_{ab}\epsilon^{rs})^2 + \frac{1}{4}W(*F^r)^2\right)$$
$$-A^r_0 \partial_c \Pi^c_r + \Lambda \epsilon^{ab}\partial_b\left(\frac{\partial_a X^M P_M + \Pi^c_r F^r_{ac}}{\sqrt{W}}\right) \quad (13)$$

where P_M are the conjugate momentum to X^M while Π^a_r are the corresponding momentum to A^r_a. The index r denote the 2 compactified directions on the target space. a is the world volume index while M label the LCG transverse directions in the target space.

Its supersymmetric extension may be obtained in an straightforward way from the supermembrane hamiltonian in the LCG by the procedure described in [30].

We may solve explicitly the constraints on Π^c_r obtaining

$$\Pi^c_r = \epsilon^{cb}\partial_b \Pi_r; \qquad r = 1, 2 \quad (14)$$

Defining the 2-form ω in terms of Π_r as

$$\omega = \partial_a \Pi_r \partial_b \Pi_s \epsilon^{rs} d\xi^a \wedge d\xi^b, \quad (15)$$

the condition of non trivial membrane winding imposes a restriction on it, namely

$$\oint_\Sigma \omega = 2\pi n. \quad (16)$$

With this condition on ω, Weil's theorem ensures that there always exist an associated $U(1)$ principal bundle over Σ and a connection on it such that ω is its curvature. The minimal configurations for the hamiltonian (13) may be expressed in terms of such connections.

In [29] the minimal configurations of the hamiltonian of the double compactified supermembrane were obtained. In spite of the fact that the explicit expression (13) was not then obtained, all the minimal configurations were found. They correspond to $\Pi_r = \hat{\Pi}_r$ satisfying

$$*\,\hat{\omega} = \epsilon^{ab}\partial_a \hat{\Pi}_r \partial_b \hat{\Pi}_s \epsilon^{rs} = n\sqrt{W} \qquad n \neq 0 \quad (17)$$

The explicit expressions for $\hat{\Pi}_r$ were obtained in that paper [29]. As mentioned before, they correspond to $U(1)$ connections on non trivial principle bundles over Σ. The principle bundle is characterized by the integer n corresponding to an irreducible winding of the supermembrane. Moreover the semiclassical approximation of the hamiltonian density around the minimal configuration, was shown to agree with the hamiltonian density of super Maxwell theory on the world sheet, minimally coupled to the seven scalar fields representing the coordinates transverse to the world volume of the super-brane.

As mention in the introduction these minima correspond to the dual solutions of section 2 corresponding to 2D-brane. We now consider the hamiltonian of the D=10 4D-brane. It may be obtained by the following double dimensional reduction procedure. We start from the PST action for the super M5-brane. We consider the gauge fixing condition which fixes the scalar field to be proportional to the world volume time. We then perform the usual double dimensional reduction by taking one of the target space coordinates $X^{11} = \sigma^5$ where σ^5 is one of the world volume local coordinates. After several calculations we end up with the following canonical lagrangian

$$\mathcal{L} = P_m \dot{X}^m + P^{ij}\dot{B}_{ij} - \mathcal{H}_c \quad (18)$$

$$\mathcal{H}_c = \lambda\phi + \lambda^i \phi_i + \theta_i \partial_j P^{ij} \quad (19)$$

where

$$\phi = \frac{1}{2}P^2 + 2g + 2\left(\frac{1}{8}P^{ij}P^{kl}g_{ik}g_{jl} + *H^i * H^j g_{ij}\right)$$
$$+ \frac{1}{32}\left(\frac{1}{4}\epsilon_{ijkl}P^{ij}P^{kl}\right)^2 \quad (20)$$

$$\phi_l = P_m \partial_l X^m + \frac{1}{2}\epsilon_{ijkl}P^{ij} * H^k \quad (21)$$

$$*H^i = \frac{1}{6}\epsilon^{ijkl}H_{jkl} \quad (22)$$

We finally obtain the hamiltonian in the LCG

$$\mathcal{H} = \frac{1}{\sqrt{W}}\left(\frac{1}{2}P^M P_M + 2g + 2\left(\frac{1}{8}P^{ij}P^{kl}g_{ik}g_{jl}\right.\right.$$
$$\left.\left. + *H^i * H^j g_{ij}\right) + \frac{1}{32}\left(\frac{1}{4}\epsilon_{ijkl}P^{ij}P^{kl}\right)^2\right.$$
$$\left. + \Lambda^{lq}\partial_q\left(P_M \partial_l X^M + \frac{1}{2}\epsilon_{ijkl}P^{ij} * H^k\right)\right) \quad (23)$$

where Λ^{lq} are antisymmetric lagrange multipliers associated to the generator of volume preserving diffeomorphisms.

There is also a global constraint given by

$$\oint_{C_i}\left(P_M\partial_l X^M+\frac{1}{2}\epsilon_{ijkl}P^{ij}{*}H^k\right)d\sigma^l=0 \tag{24}$$

where C_i is a basis of homology of dimension 1. We will not consider any further this global constraint. We will work only with the diffeomorphisms connected to the identity.

We now consider a target space with 4 compactified directions, $M_6 \mathrm{x} S^1 \mathrm{x} S^1 \mathrm{x} S^1 \mathrm{x} S^1$. We construct the dual formulation associated to the compactified directions. We associate to each X_r, $r=1,...,4$ a B^3 3-form

$$dX_r \to dB_r^3 \tag{25}$$

It is more convenient to work with the Hodge dual of the 3-form,

$$B_{rijk} \to A_r^l \tag{26}$$

in the spatial world volume sector B_{rijo} are Lagrange multipliers. We denote Π_{rl} the conjugate momenta to A_r^l.

There is a constraint on Π_{rl} which yields

$$\Pi_{rl}=\partial_l\Pi_r \tag{27}$$

We may then perform a canonical transformation to obtain

$$\Pi_{rl}\dot{A}_r^l=\dot{\Pi}_r(\partial_l A_r^l) \tag{28}$$

that is, $\partial_l A_r^l$ is the conjugate momenta to Π_r:

$$\begin{aligned}\Pi_r &\equiv \mathcal{A}_r\\ \partial_l A_r^l &\equiv \mathit{\Pi}_r\end{aligned} \tag{29}$$

We then obtain the dual formulation to (23).

For the determinant of the induced metric we obtain

$$\begin{aligned}&\frac{1}{4!}\left(\epsilon^{i_1\ldots i_4}\partial_{i_1}X^{a_1}...\partial_{i_4}X^{a_4}\right)^2\\
&\to\frac{1}{4!}\left(\epsilon^{i_1\ldots i_4}\partial_{i_1}X^{b_1}...\partial_{i_4}X^{b_4}\right)^2\\
&+\frac{1}{3!}\left(\epsilon^{i_1\ldots i_4}\partial_{i_1}\mathcal{A}_r\partial_{i_2}X^{b_2}...\partial_{i_4}X^{b_4}\right)^2\\
&+\frac{1}{2!}\left(\epsilon^{i_1\ldots i_4}\partial_{i_1}\mathcal{A}_r\partial_{i_2}\mathcal{A}_s\partial_{i_3}X^{b_3}\partial_{i_4}X^{b_4}\right)^2\\
&+\left(\epsilon^{i_1\ldots i_4}\partial_{i_1}\mathcal{A}_r\partial_{i_2}\mathcal{A}_s\partial_{i_3}\mathcal{A}_t\partial_{i_4}X^{b_4}\right)^2\\
&+\left(\epsilon^{i_1\ldots i_4}\partial_{i_1}\mathcal{A}_r\partial_{i_2}\mathcal{A}_s\partial_{i_3}\mathcal{A}_t\partial_{i_4}\mathcal{A}_u\right)^2\end{aligned} \tag{30}$$

where the index b is used to denote the non compactified directions.

For the terms quadratic on the momenta to the antisymmetric field B_{ij} we obtain similar terms, the connection 1-forms $\mathcal{A}_r$ replaces the corresponding terms where the compactified coordinates appear. In the same way the momenta $\mathit{\Pi}^r$ replaces the conjugate momenta of the compactified coordinates. That is, in the dual formulation (with the additional canonical transformation we mentioned above) the compactified coordinates are replaced by the connection 1-forms $\mathcal{A}_r$. However, it is clear from the dual formulation that $\mathcal{A}_r$ may have non trivial transitions of a very specific form over a nontrivial bundle. This fact is difficult to realize in terms of the original maps from the world volume to the compactified directions of the target space.

4. The noncommutative formulation

We now introduce a symplectic 2-form in the previous formulation. We take

$$F_{ij}d\sigma^i\wedge d\sigma^j \tag{31}$$

where F_{ij} is the curvature of the connection 1-form which minimize the hamiltonian of the theory. For the D=11 Supermembrane we obtained

$$*F=n, \qquad i,j=1,2 \tag{32}$$

as discussed before.

For the D=10 Super 4D-brane we obtain

$$F\propto *F \tag{33}$$

$$*(F\wedge F)\propto n \tag{34}$$

over the 4 dimensional spatial world volume. These are two of the dual solutions introduced in [31] and explained in section 2.

The procedure to obtain the symplectic noncommutative formulation for the double compactified D=11 Supermembrane was explicitly introduced in [30]. We will now obtain a similar formulation for the 4 D-brane in 10 dimensions.

The approach uses the symplectic 2-forms previously introduced to obtain a description of the theory in terms of symplectic connections over a symplectic fibration.

We consider the metric W^{ij} defined by

$$W^{ij} \equiv \hat{\Pi}^i_r \hat{\Pi}^j_r \tag{35}$$

where

$$\hat{\Pi}^i_r \equiv F^{ij} \partial_j \hat{\mathcal{A}}_r \tag{36}$$

The metric W^{ij} is taken to be the metric over the spatial world volume for which F^{ij} satisfies the duality conditions. $\hat{\Pi}^i_r$ is a well defined vielbein. (36) defines $\hat{\mathcal{A}}_r$.

We then introduce the following Poisson bracket over the world volume

$$\{B, C\} \equiv F^{ij} \partial_i B \partial_j C \tag{37}$$

It satisfies the Jacobi identity

$$\begin{aligned} &\{\{A, B\}, C\} + \{\{C, A\}, B\} + \{\{B, C\}, A\} \\ &\quad = \left(F^{kl} F^{ij} + F^{jl} F^{ki} + F^{il} F^{jk}\right) \\ &\qquad .D_l \left(\partial_i A \partial_j B \partial_k C\right) \\ &\quad = k \frac{\epsilon^{klij}}{\sqrt{W}} D_l (\partial_i A \partial_j B \partial_k C) = 0 \end{aligned} \tag{38}$$

Where D_l denotes the covariant derivative with respect to the metric W^{ij}. A, B and C are scalar fields.

We now introduce the rotated covariant derivative

$$D_r \equiv \hat{\Pi}^i_r D_i \tag{39}$$

$$\mathcal{D}_r = D_r + \{\mathcal{A}_r, \} \tag{40}$$

and curvature

$$\mathcal{F}_{rs} = D_r \mathcal{A}_s - D_s \mathcal{A}_r + \{\mathcal{A}_r, \mathcal{A}_s\} \tag{41}$$

The hamiltonian density of the double compactified Supermembrane was expressed in terms of these geometrical objects in [30]. We now perform the analogous formulation for the 4 D-brane.

We obtain

$$\begin{aligned} &\frac{1}{4!} \left[\epsilon^{i_1 \ldots i_4} \partial_{i_1} X^{b_1} \ldots \partial_{i_4} X^{b_4}\right]^2 \\ &\rightarrow \frac{1}{4!} \left[\{X^{b_1}, X^{b_2}\}\{X^{b_3}, X^{b_4}\}\right]^2 \end{aligned} \tag{42}$$

$$\begin{aligned} &\frac{1}{3!} \left[\epsilon^{i_1 \ldots i_4} \partial_{i_1} \mathcal{A}_r \partial_{i_2} X^{b_2} \ldots \partial_{i_4} X^{b_4}\right]^2 \\ &\rightarrow \frac{1}{3!} \left[\mathcal{D}_r X^{b_2} . \{X^{b_3}, X^{b_4}\}\right]^2 \end{aligned} \tag{43}$$

$$\begin{aligned} &\frac{1}{2!} \left[\epsilon^{i_1 \ldots i_4} \partial_{i_1} \mathcal{A}_r \partial_{i_2} \mathcal{A}_s \partial_{i_3} X^{b_3} \partial_{i_4} X^{b_4}\right]^2 \\ &\rightarrow \frac{1}{2!} \left[\mathcal{F}_{rs} . \{X^{b_3}, X^{b_4}\}\right]^2 \end{aligned} \tag{44}$$

$$\begin{aligned} &\left[\epsilon^{i_1 \ldots i_4} \partial_{i_1} \mathcal{A}_r \partial_{i_2} \mathcal{A}_s \partial_{i_3} \mathcal{A}_t \partial_{i_4} X^{b_4}\right]^2 \\ &\rightarrow \left[\mathcal{F}_{rs} \mathcal{D}_t X^b\}\right]^2 \end{aligned} \tag{45}$$

$$\left[\epsilon^{i_1 \ldots i_4} \partial_{i_1} \mathcal{A}_r \ldots \partial_{i_4} \mathcal{A}_u\right]^2 \rightarrow \left[\mathcal{F}_{rs} \mathcal{F}_{tu}\right]^2 \tag{46}$$

where it is understood that the antisymmetric part on the b indexes and the target space r, s,... indexes is taken.

Similar formulae may be written for all other terms in the hamiltonian density for the 4 D-brane. They may be rewritten in terms of the bracket (37), the covariant derivative and the curvature $\mathcal{F}_{rs}$.

The expression for the double compactified D=11 Supermembrane was [30]:

$$\begin{aligned} H &= \int_\Sigma \mathcal{H} = \int_\Sigma \frac{1}{2\sqrt{W}} \left[(P^M)^2 + (\Pi^r)^2\right. \\ &+ \frac{1}{2} W \{X^M, X^N\}^2 + W (\mathcal{D}_r X^M)^2 \\ &\left. + \frac{1}{2} W (\mathcal{F}_{rs})^2\right] + \int_\Sigma \left[\frac{1}{8} \sqrt{W} n^2\right. \\ &\left. - \Lambda \left(\mathcal{D}_r \Pi^r + \{X^M, P_M\}\right)\right] \end{aligned} \tag{47}$$

The geometrical interpretation of the above formulation (47) was given in [30] in terms of symplectic fibrations and connection 1-form over it. We have shown here that the same geometrical description may be given for the 4 D-brane. We will discuss it in more detail shortly. Before then, we would like to remark that there is a natural deformation of the above formulations in terms of the Moyal bracket. Let us see how this deformation may be realized preserving the symplectic structure on the fibration, for the case of the double compactified Supermembrane.

We replace in (47) the Poisson bracket

$$\{B, C\} = F^{ij} \partial_i B \partial_j C \tag{48}$$

by the Moyal bracket $\{,\}_M$. (48) is the first term in the expansion of the Moyal bracket. We notice that under the gauge transformation generated by the first class constraint

$$\delta X^M = \{\xi, X^M\} \tag{49}$$

$$\delta \mathcal{A}_r = -\mathcal{D}_r\xi = -\left(D_r\xi + \{\mathcal{A}_r, \xi\}_M\right) \tag{50}$$

$$\delta \mathcal{D}_r X^M = \{\xi, \mathcal{D}_r X^M\}_M \tag{51}$$

These properties ensure that the hamiltonian density transform as

$$\delta \mathcal{H} = \{\xi, \mathcal{H}\}_M \tag{52}$$

The integral over a compact world volume renders the canonical lagrangian invariant under the gauge transformations.

We notice that the physical degrees of freedom of both theories, the double compactified D=11 Supermembrane (47) and its Moyal deformation, are exactly the same. Moreover it can be shown that there is a one to one correspondence, in the sense of Kontsevich, between both theories.

In [30] a geometrical description of the symplectic non commutative gauge theory was introduced. The same geometrical interpretation may be used to describe its deformation in terms of the Moyal bracket and the compactified 4 D-brane we have discussed.

We consider a symplectic fibration with base manifold the spatial world volume, which is a closed (without boundary) manifold. Over the fibration we consider the (infinite dimensional) Poisson bracket of the sections $X^M(\sigma)$, $P_M(\sigma)$:

$$[X^M(\sigma), P_M(\sigma')] = \delta(\sigma, \sigma') \tag{53}$$

This Poisson structure is preserved under the transition maps of the fibration. These maps are defined by

$$\delta X^M = \{\xi, X^M\} \tag{54}$$

$$\delta P_M = \{\xi, P_M\} \tag{55}$$

over $U_\alpha \bigcap U_\beta$, U_α is a covering of the base manifold. We notice that the transformation maps are defined in terms of the (finite dimensional) Poisson bracket or Moyal bracket over the world volume, and they preserve the (infinite dimensional) Poisson bracket on the fibration. $\mathcal{A}_r$ define a symplectic connection over the fibration. That is, the Poisson bracket on the fibration is preserved under the holonomy generated by $\mathcal{A}_r$.

The three theories we have discussed, the double compactified D=11 Supermembrane, its Moyal deformation and the compactified 4 D-brane all admit the same geometrical interpretation. They describe the dynamics of a symplectic connection over a symplectic fibration.

5. Conclusions

We formulated the double compactified D=11 Supermembrane and the compactified Super 4 D-brane in terms of a symplectic noncommutative gauge theory. We constructed a deformation of the compactified D=11 Supermembrane in terms of the Moyal brackets. There exist a one to one correspondence between both theories in the sense of Kontsevich. A unified geometrical description of these theories was given in terms of a symplectic fibration over the world volume and the dynamics of symplectic connections over it. We hope the analysis for the D=10 4D-brane may be extended to describe the M5-brane in 11 dimension. Once that formulation is available we may start analysing the corresponding quantum field theory. We hope to report on that shortly.

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